APPLICATION OF ANALOG COMPUTERS TO INVENTORY CONTROL PROBLEMS

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B. S. E. E., Robert College, Istanbul, 1964

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

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ACKNOWLEDGMENTS

The writer wishes to acknowledge the aid of his major professor, Professor John Clifton, who provided the initial inspiration and continual guidance on the thesis.

INTRODUCTION

One of the main problems of industrial engineering is finding the optimum operating point or the optimum operating policy of a system. Most systems studied are incompletely structured. These systems operate in such a way that the output for a particular input can only be determined in probablistic terms. The control and feedback mechanisms are not very reliable and do not operate in the same way. Since most familiar optimization methods cannot be used for incompletely structured systems, a deterministic method is required for problems of varying characteristics. Systems analysis fulfills these requirements and can be applied to a wide range of problems.

The first step in any systems analysis approach is to determine the components. Then the system boundaries which define the extent of the analysis must be set up. Incorrect boundaries may result in incorrect solutions. The next step is to build a physical or a mathematical model to represent the system. By simulating input conditions, resulting outputs may be noted. Input and other conditions under control of the analyzer may be varied until the required output is obtained. For an existing system, a more economical operating point may be found by using systematic search techniques and eliminating non-feasible alternatives. Thus a model may be used to find the characteristics, operating conditions and optimum operating policy of a system.

An important point to recognize in system analysis is that the model must be a realistic representation of the system. Otherwise, results will have no meaning. Results of a simulation model should not be taken as final but must aid management in making decisions.

Models

A model is any technique, device or process by means of which specific relationships of the system characteristics can be investigated. Two kinds of models which can be used in the study of systems are: (a) mathematical models and (b) analog models.

Mathematical Models. Mathematical models used mathematical functions to represent the system. The system's input may be assumed to be this function. The processor, acting on the function, modifies it to give the required output function. The action on the input may be integration, differentiation, addition or any one of the mathematical operations that is possible to perform. It is also possible for the system to consist of various subsystems. The processors then will give an output according to their own characteristics, each being different from the other. But in most cases, a mathematical relationship can be found which represents the over-all mathematical operation that is applied to the input. This gives rise to the transfer function concept. This concept represents the total or simplified mathematical functions which act on the input function to give the output function. This is a great simplification for mathematical models but sometimes the over-all transfer function is so complicated that it is not practical to use or to even attempt to derive.

The main problem encountered in constructing a mathematical model is in representing certain system parts as mathematical functions or operations. It is difficult to build a mathematical model, especially with scheduled systems. For incompletely structured systems, it may be impossible to build the model. In this system, the human has some important roles that nobody has yet been able to represent in exact or even approximate mathematical terms.

When exact mathematical relations cannot be used, statistical methods may be used with the mathematical relations. Since human behavior is difficult to predict, statistical methods may help represent the most likely behavior while concurrently making some allowance for erratic behavior.

The only disadvantages of the mathematical model are that it introduces too many variables and that it greatly complicates the analysis. An over-all transfer function may be impossible to derive. When this model becomes too complicated, it is better to use analog models.

Analog Models. Analog models are physical representations of systems. They are usually smaller than the actual model and are cheap to build and operate. The analog model, by allowing changes in different components, makes possible a very good study of the system under a wide variety of conditions (6). For example, airplane designers learn much about the characteristics of a new airplane by testing a model in a wind tunnel. Analog models play a very important role in servomechanism design. One characteristic of these models is the ease of simulation; results are obtained more quickly and are easier to interpret than those obtained from mathematical models. Analog models offer a very easy method of studying the system under varying conditions. The use of analog models will help locate the optimum operating point by searching through a number of possibilities and selecting the best combination.

With proper scaling, the simulation will enable the experimenter to get his result in real time whereas this is expensive to do with mathematical models. Thus, various possibilities can be examined in a relatively short time and the outputs of each can be studied to help locate the optimum. Proper time scaling will give the total time needed to complete the operation under investigation.

A complicated system may consist of a number of subsystems, each having its own characteristics and operations requirements. If each subsystem is designed for optimum performance, the resultant output of the entire system may be far from optimum. Models of subsystems are very efficient tools in overcoming this difficulty. In both the mathematical and the analog models, the complete system may be optimized by making trade-offs between the subsystems. Trade-offs are very easy in analog models. In mathematical models, a slight change in one of the subsystems will require a new derivation of the over-all transfer function for the system. This can be very tedious if a large number of possibilities are to be studied. With an analog model, changing a few components and then activating the model will not take long and is not complicated.

The accuracy of the analog model depends on the accuracy of the components used in building it. The mathematical model may have better accuracy if the calculations are made with proper care. No matter what the accuracy of the analog model, the results are extremely helpful to the engineer.

PROBLEM

Analog Computers

An analog computer is a device which deals with physical quantities representing numerical values. Individual operation blocks, mechanical or electronic in nature, are coupled together to establish a physical system whose behavior is similar to that of the problem under study.

Operational Amplifier. Most electronic analog computer operations are done with the aid of an operational amplifier. Usually it is a DC amplifier of high gain and high input impedance with suitable external connections. These amplifiers can be made to add, subtract, differentiate or integrate.

Input and Output Relations of DC Amplifiers. Let A be the gain of the operational amplifier, e_g be the input voltage and e_0 be the output voltage. Input and output are related by the following equation:

$$e = -A\ e_g \qquad \text{(Eq. 1)}$$

Figure 1 shows a DC amplifier.

Scalar Multiplication Operation. If input resistance R_1 is added between the input and the amplifier and resistance R_f is used to provide the feedback, it can be shown that the amplifier inverts the input voltage and multiplies it by a constant. Figure 2 is an illustration of this.

Addition. By using several separate inputs, the operational amplifier may be made to add. Amplifier connections for addition is shown in Fig. 3.

Subtraction. If some of the input voltages have a different polarity from the others, then the operational amplifier may be made to subtract. Figure 4 shows the subtraction connections.

Differentiation. The operational amplifier can be used to differentiate by using a capacitive input and a resistive feedback element. The amplifier is seldom used for this purpose since the noise in the input tends to be magnified by differentiation whereas in integration it tends to be cancelled. Also, the differentiation circuits tend to be unstable. A differentiation circuit is shown in Fig. 5.

Integration. If a capacitor is used as a feedback element, the operational amplifier will act as an integrator. Figure 6 shows the connection for an integration circuit. The derivation of the input and the output relations for an integration circuit is given below:

$$\frac{e_i}{R_i} = i = \frac{dQ}{dt}$$

$$dQ = C_f\ de_o$$

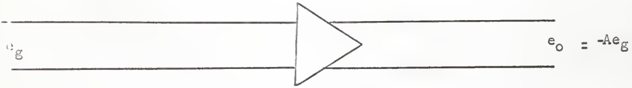


Fig. 1. Input and output relations of a DC amplifier.

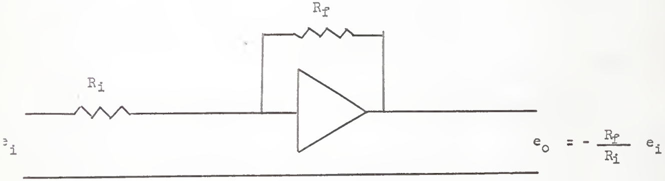


Fig. 2. Operational amplifier for scalar multiplication operation, with ground connection shown.

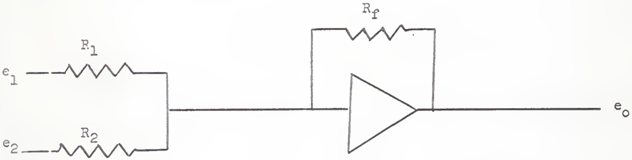


Fig. 3. Operational amplifier for addition; ground circuit omitted.

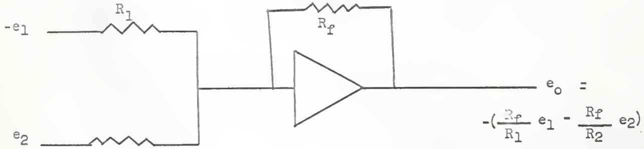


Fig. 4. Subtraction on an operational amplifier.

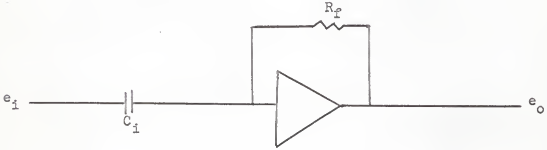


Fig. 5. Differentiation on an operational amplifier.

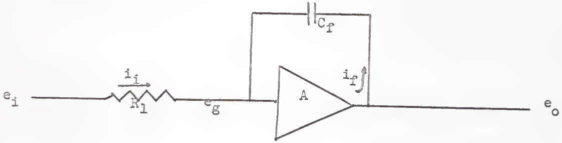


Fig. 6. Integration on an operational amplifier.

$$\frac{e_i}{R_i} = -C_f \frac{de_o}{dt}$$

$$de_o = -\frac{1}{R_iC_f} \; e_i dt$$

$$e_o = -\frac{1}{R_iC_f} \int_t e_i dt + e_o$$

Initial Conditions. In most cases, the programmer must take care of initial conditions of the system. These represent the condition of the system at starting time T_0. In an integration circuit, the initial conditions are applied in form of charges on the capacitor and stored before the operation was started.

Other characteristics of the analog computers are that they measure continuously and that the operation can be stopped at any time and then resumed from the point of stop. One powerful application of analog computers is simulation in which physical properties, not easily varied, are represented by voltages which are easily varied. In practice, factors affecting the operation of the system cannot be changed quickly but, on the computer, any one or all of these may be varied at will and the results observed as the changes are made.

Inventory Control as a System

Inventory control problems are particularly well adapted to system analysis and, correspondingly, to further studies using the model concept. Although most industrial engineering problems are well-suited to systems analysis, generally there is only one applicable model. In inventory control problems, both the mathematical and the analog models can be set up. The classical approach is to set up a mathematical model and then study it under different conditions. The result is an operating doctrine and a set of decision rules.

This thesis attempts to set up an analog model for inventory control systems. By using the analog computer to set up the model, it is hoped that a relatively inexpensive way of overcoming difficulties in this field can be found.

By applying system analysis concepts to the inventory control systems, the five important system components can be identified easily. They are: (1) Input - representing the material that is obtained from the manufacturer or the material that is being manufactured for inventory; (2) The processor - the combination of techniques and methods used to control the inventory; (3) Feedback mechanism - the customer reaction, such as customer satisfaction; (4) Control mechanism - the policy of the firm with respect to the operation policy; and (5) Output - the material that is being supplied to the customers.

In the mathematical model, it is customary to attach some financial penalties, such as stockout costs, to feedback and control elements. Storage cost and order costs are always indicated and the optimum operation point is found by either breaking even with the cost, the maximization of profit or the minimization of losses.

Penalties used for these calculations cannot be evaluated with a great degree of accuracy. Therefore, the best that can be obtained from a mathematical model is an approximate solution, the accuracy being determined by the accuracy of the penalties.

The main target of the system analysis is the optimization of the system operation. Therefore, the aim will be to optimize the operation of the inventory systems, i.e., optimization of the customer services.

It was mentioned that the mathematical model is optimized by either maximizing the profit, minimizing the losses or by breaking even. Although the analog model is still concerned with cost, the optimum operating point will be the point which satisfies the customer and the firm when the system is performing under the specific constraints set by the policy of the firm. This, in turn, means that the level of the inventory must be such that the demand is satisfied within the limits that have been set. Satisfactory limits are set up by the characteristics of the system processor.

General System Characteristics

There are as many inventory control systems as there are business and manufacturing firms. Every time some material is stored with anticipation of meeting future demand, such a system is created. Any method used to control or organize the flow of material in and out of storage can be analyzed by the systems approach. The criteria for the effectiveness of each inventory control system can be measured in terms of the customer's satisfaction or dissatisfaction. It is very hard to measure satisfaction. To overcome this difficulty, a standard called service criterion is used to measure the effectiveness of the system with respect to customer satisfaction. Service criterion is the permitted number of stockouts during a given number of cycles. A service criterion of ten will indicate that ten stockouts during the total number of cycles to be studied is permitted. On the other hand, a service criterion of 0 indicates that no stockouts can be permitted during the system operation time. This is based on a probability of one change in 1,000,000.

The service criterion may be found by using the various constraints that are effective on either the processor or on the input. These constraints depend on factors such as the availability of capital, the penalty for losing a sale, the penalty for losing a customer and others.

Many methods can be suggested for calculating the service criterion. The expected number of stockouts can be calculated by using a purely statistical approach, or a forecast of the situation can be made using the Time Series Analysis Method. If each system is in operation for a number of cycles, then past data can be used to obtain the number of expected stockouts during the coming period.

A simple method is proposed here. It will be assumed that when a sale is lost, the profit that could have been realized is also lost. In addition, a certain percent of the customers will go elsewhere to satisfy their demand and will never come back. So the cost of a stockout may be represented as a function of the lost profit and the lost customers. Very simply, the stockout costs will be represented as:

$$S = (N-n)\,P(k-1) + NP \qquad \text{(Eq. 5)}$$

where S is the stockout cost, N is the number of customers whose demand cannot be met, n is the number of customers who will wait for the next cycle, k is the number of cycles in one period, and P is the profit made on one unit of material.

It is assumed that the loss of the sales affects the firm only for one time period. New customers can be found or a new product might induce the lost customer to come back.

After the stockout cost is calculated, the next step is to calculate the service criterion. Every firm has set a limit on the amount of loss that can be sustained by a stockout (6). The division of this amount by the total stockout costs will give the value of the service criterion, represented by the following equation:

$$S.C = L/S \qquad \text{(Eq. 6)}$$

where S.C is the service criterion, L is the stockout loss that can be accepted by the firm and S is the stockout cost calculated from the previous formula. The economic system operation is discussed in preceding paragraphs.

Service criterion plays a very important role in analog models. It will be used as a constraint on the total number of permitted stockouts. Two possible applications of the service criterion concept are:

(a) The permitted number of stockouts may be determined and, during a simulation run, care must be taken not to exceed this number, and

(b) The given system may be simulated for actual operation with randomly varying demand and the number of stockouts may be determined for a constant replenishment rate of material placed in inventory.

In both cases, the input conditions may be altered so that the over-all performance characteristics will be satisfactory. The system will perform according to the service criterion. Both methods are easy to handle. In this thesis, the second method will be used.

As was explained, the service criterion is a measure of the system's effectiveness. The prime aim of the system is to meet operating conditions imposed by the service criterion. Therefore, the problem becomes that of determining the amount of material to be placed in storage. This depends on four factors in addition to the service criterion: (1) The frequency with which demands occur; (2) The distribution of demand; (3) The amount demanded during one cycle; and (4) The rate at which replenishment occurs.

Identifying the system components once more results in: Input - replenishment; Processor - supply system; Output - amount of material required to meet demand; Feedback - customer satisfaction expressed in terms of service criterion; Control - system constraints, such as economic situation and the amount of available space.

Using the above information, a satisfactory storage level can be found. Any decision which is not the true decision will result in excessive material in either storage (overstock) or a stockout case.

Among the factors affecting the operation of the system, the output of the system occupies a very important place. Since human behavior is hard to predict, it is impossible to determine exact demand figures. A perfect forecast of demand requires insight into the future. Since this is impossible, demand is always an estimate which can vary from the calculated value. Most of the time, demand is forecast between two values, a maximum and a minimum. A negative change in demand will result in increasing the amount of material in storage. This can be remedied by decreasing the input rate; less material will be placed in inventory until the excess stock is used. But if change in demand is positive, i.e., more material is demanded, it is possible to run out of material and not be able to meet the demand. This situation cannot be remedied very quickly by changing the input conditions since there is generally a time lag between the time of notification for an increase in the input rate and the actual increase.

An ideal situation would require an infinite amount of material in storage. All demands may now be met and there will not be a single case of stockout. However, this is the most uneconomical situation that can be conceived. An infinite amount of material and an infinite amount of storage space would be required.

Besides other costs of maintaining an inventory, capital invested in material to be stored represents a possible loss of profit that could be realized if invested at some other place. A practical limit has to be set for the amount invested in stored materials. From the investor's point of view, the total investment must be kept as low as possible in order to get maximum returns on the investment. Therefore, the investor will want to have the minimum amount of material that will give satisfactory customer service. He will carry a certain amount of material in stock and replenish it as the stock is depleted. His ideal condition is to have zero units at the end of each cycle. As this point is reached, he must replenish the material so the maximum inventory level is reached at the beginning of the next cycle. This process is continued as long as there is demand for that particular material.

If the demand is constant, this technique of achieving the ideal condition would be very simple. Each time the inventory reaches a certain level, an order will be placed for more material. Another way would be to place an order for more material at a preassigned date. But since the demand is never known, the ideal condition does not occur and the inventory control system, in order to cope with this difficulty, must be very complex.

With a simple and inexpensive analog model, a variety of cases can be solved or studied. Before starting the construction of an analog model, system boundaries have to be set up.

Boundaries. Although each inventory control system is unique with respect to its general characteristics, very general boundaries could be set up by limiting the possibilities of action.

There are three kinds of inventories: (1) Raw material; (2) In-process; and (3) Finished materials.

It is possible that each has constraints which are not common to the other. For example, a tomato (raw material) cannot be stored indefinitely but canned tomatoes (finished material) can be stored indefinitely without any special care. It will be assumed that only one kind of inventory is treated in each model and the interactions between the others will not be considered. All three cases may be studied separately with their own initial constraints.

Engineering design changes may make a material obsolete. It is not good practice to carry material in storage which has been made obsolete by newer and better products. Here it will be assumed that there will be a demand for every material that is in storage and that the passage of time will depreciate the value of the material. This obsolescence cost is included in the carrying cost.

Product mix situations will not be studied using a single model. One product at a time will be studied. Each product can be examined separately. A common constraint resulting from the mix situation may be used in separate studies. The analog model to be discussed here will consider only one product at a time.

In short, the system to be studied will consist of an input which will be a single commodity, product or material. The input is in any one of the three inventory classes and is one which will not become obsolete.

The environmental factors that might affect the material in stock will be ignored. It will be assumed that satisfactory conditions exist for the storage of the material in question.

Boundaries, which are explained above, are used to clearly define the system and the extent of the analysis. This definition will help in the selection of the input and the output conditions in the simulation and in making the model more realistic. Together with the constraints, the boundaries are used to pinpoint the problem, thus making sure the right problem is studied in a realistic fashion.

Constraints. A constraint is a limiting factor. In inventory control problems, there are many constraints which have very important roles in the operation of the system. At the time of the study of the system, there may be many constraints which are unknown and unsuspected.

The number of constraints must be kept to a minimum for accurate results in analog models. If too many are used, the analyst may be forced to ignore some possible alternatives. When alternatives are limited, the accuracy of the model diminishes because some alternatives were never considered.

Replenishment of Stock

Stock replenishment can take place either instantaneously or finitely. In instantaneous replenishment, all material to be placed in storage is delivered in one batch or shipment. No time is lost in handling. Very small replenishments may be treated as instantaneous replenishment.

In finite replenishment, a constant rate of delivery continues until all material is in storage. This might mean two or more shipments at different times. Figure 7 is a graphical representation of the two replenishment methods. The lag between the time of order and the time of receipt of the order is discussed under the heading "Lead Time."

In the graph representing the instantaneous replenishment rate, it can be seen that all material was placed in storage at time 't', the first and the last units being placed at the same time.

In the other case, the first unit is placed in storage at time t_1 and the last unit is placed at t_2. The time lag between the first unit and the last unit is $(t_1 - t_2)$ time periods.

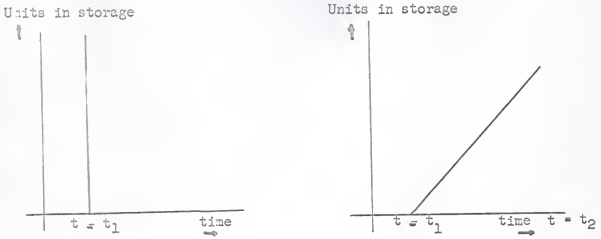
Although finite replenishment is represented by a straight line, there is always a possibility of irregularities in the replenishment rate. The replenishment rate shown by a line might consist of small shipments arriving at random. The number of units in each shipment may vary. Figure 8 shows this situation graphically. A way has to be found so the variations and the irregularities can be approximated by a straight line or a curve.

A curve can be fitted through the points. This will not give the true value of the number of units of material remaining in storage at different times but will represent the average number of units in storage. If a straight line is fitted through the points, a true representation of the initial and the final values will be obtained.

In order to increase the accuracy of finite replenishment, a least square curve may be fitted. In inventory control, the maximum and minimum inventory levels are of utmost importance. If there are no large fluctuations or discontinuities in the replenishment methods, then it is adequate to fit a straight line and take the average of the minimum and maximum inventory levels.

If demand is continuing during the replenishment time, the slope of the replenishment curve will change. If a straight line is used to represent the replenishment curve, this change will be compensated for by changing the slope of the line. This change may be calculated and the maximum inventory level can be found easily. Figures 9 and 10 represent cases where there is demand for the material and where there is no demand for the material during replenishment.

If material is taken out of storage at the same time replenishment is continuing in order to meet demand, then the slope of the replenishment line decreases. If (γ) is the replenishment per unit cycle (maximum number of units placed in storage, (λ) the unit cycle demand and (Q) the size of the lot produced, then the slope of the replenishment line will be (γ),



Instantaneous rate

Finite rate

Fig. 7. Replenishment rates.

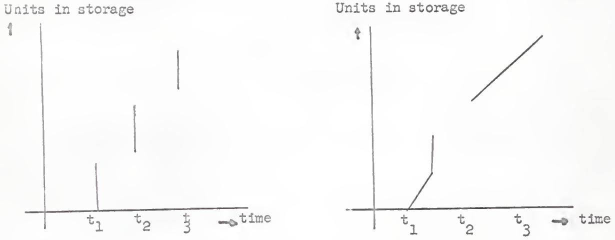


Fig. 8. Irregularities in finite replenishment rate.

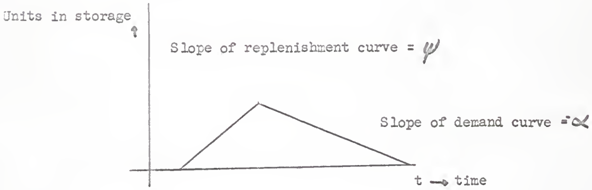


Fig. 9. No demand during replenishment.

indicating the flow of material into the warehouse without any demand occurring at the same time. $(\psi - \lambda)$ will be the slope of the replenishment line if demand is occurring during replenishment.

For some inventory systems, an assumption is that the material sold will be treated as a continuous variable instead of discrete units. In Fig. 11, the discrete replenishment is shown.

This causes no difference in the treatment of the problem. Especially when the analog model is used, the discrete case has to be converted to a continuous case. Therefore, continuous replenishment will be used for all cases. There is no major difference between these two replenishment modes since at time 't' the amount of material will be the same in both cases. The maximum inventory level at the end of the replenishment will be the same regardless of the replenishment type.

There are some systems where replenishment is continuous. For example, gasoline storage problems can be studied only if continuous variables are used. If gasoline is pumped into a storage tank, the amount of material is not increasing in units but is increasing continuously. There are many other materials that can be treated similarly. Therefore, the assumption made is not unrealistic for all materials.

Lead Time

When an order is placed, some time passes before it can be acted upon. This time lag, called lead time, depends on the distance the material has to be transported and the delays that occur while the order is being processed.

In studying inventory systems, this time lag must be considered and also must be integrated with the control mechanism. If the demand for the material increases beyond the capacity of the system, a corresponding increase

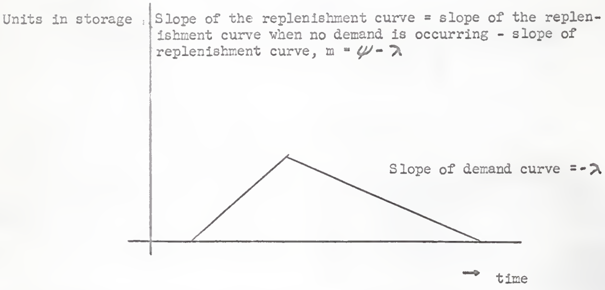

Units in storage
Slope of the replenishment curve = slope of the replenishment curve when no demand is occurring - slope of replenishment curve, m = ψ - λ
Slope of demand curve = - λ
→ time

Fig. 10. Demand during replenishment.

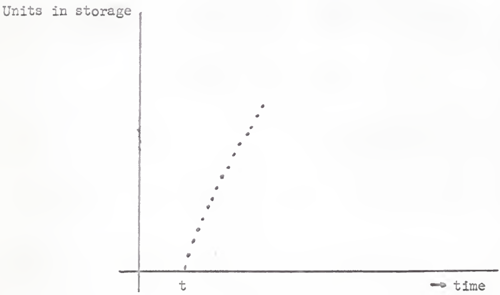


Fig. 11. Discrete replenishment.

may be required in the replenishment rate. But, because of lead time, the needed increase may not take place for a number of time periods during which the inventory system may complete a few cycles of operation (successive stocking and sales). The number of time cycles that occur before the replenishment rate is actually changed depends on the lead time. The system will not be able to function according to the lead time since the replenishment rate will not be changed for a certain period whereas demand changes occur instantaneously. There will be a deficiency of material in storage until the increased replenishment rate takes effect. Likewise, a decrease in demand may have to be corrected by a decrease in the replenishment rate which may not become effective until a certain time interval elapses. Before the change in replenishment rate takes place, the material will arrive according to the previous rate, which is higher than the corrected rate.

The fact that lead time depends on many different and incalculable factors makes it difficult to determine. An unexpected strike or damage done to a shipment may double or even triple the normal lead time. It is very important to test the model for lead time variations and note how it will behave for transitions due to those variations. The aim is to determine if the introduced lag will drastically affect the operation of the system before the replenishment rate is changed.

If the lead time variations are such that the number of permissible stockouts is exceeded during the period of study, a buffer solution must be created either by shortening the cycle time slightly or by increasing the replenishment rate. By using the analog model, numerous lead time variations may be studied and the correct operating policy found.

The manufacturer or the firm having an inventory system has effective control over the number of units placed in storage. Even if the material is to be brought from some place else, the number of units to be ordered is under close control of the manager responsible for the inventory system. Various reasons may make changes in input conditions and may be the result of lead time variations. If these variations are to be taken into account, the operations will not be affected if a buffer stock exists. If delay occurs in the delivery of material, sooner or later the material ordered will be placed in storage.

Model of Inventory Control Systems

A forecast for demand, based on past sales data, must be made for every inventory system. If such figures are not available, a way has to be found to calculate the number of units to be demanded in the future. Forecasting is a complicated task in itself. Since the main problem to be discussed here is the inventory control system and its model, the demand will be assumed to be known within two limits, a maximum and a minimum number of units that may be demanded. The demand will be varied randomly between these limits. If the distribution of the demand is known, Monte Carlo simulation methods may be used to obtain variations in the demand between the two values. If the probability distribution of the demand is not known (indicating a case of uncertainty), this may have to be converted into a problem under risk, which can be done using the Bayes/Laplace method (13). Various levels of demand will be assumed to have the same probability of occurrence. A value may then be chosen for the number of units to be demanded. This value can then be used as the demand for that particular simulation exercise.

The amount of material to be ordered in each cycle is a very important decision. The lot size is usually calculated by equating two costs connected with the ordering and stocking of material. These costs are:

(a) The order cost which is the cost of processing the order through the accounting and the purchasing departments. The labor cost of the secretaries and telephone and posting bills are included in this category. If the material is manufactured internally, then the set-up costs should also be included. This is a fixed cost which does not vary with the order size. The same amount of money and time will be spent if one or several units are reordered.

(b) Inventory carrying costs are made up of taxes to be paid on the stored material and the cost of losing interest on the money invested in the material. Obsolescence costs can also be included in carrying costs. From the above, it is obvious that inventory carrying costs are directly proportional to the amount of material in storage.

Ordering costs must be known to calculate the most economical lot size, the demand and the replenishment rate. Usually the ordering cost is given as a certain fixed sum in dollars. Carrying costs are expressed as a percentage of the unit cost of the material. Using the formula below, the most economical lot size can be calculated for a constant demand with constant replenishment.

$$q = \frac{2\,AS}{I} \qquad \text{(Eq. 7)}$$

where q is the economical lot size, A is the order cost in dollars, s is the yearly demand in units per year and I is the carrying cost expressed as a percentage of the unit cost.

If the replenishment rate is finite, the following formula is to be used:

$$q = \frac{2\ AS}{I\ (1 - s/p)} \qquad \text{(Eq. 8)}$$

where p is the production or replenishment rate; the rest of the variables are as explained in the previous paragraph.

Both formulas are based on a single demand figure for the calculation of the minimum cost order quantity size. This is not realistic because a single demand figure is not known. Therefore, after this basic step, uncertainties must be eliminated from the calculations.

A way has to be found to integrate the uncertain nature of the demand with the calculations. This can be done by chosing the demand level randomly between the two limits of demand. After a time period is studied, the demand value may be changed and another value used for the calculations. If a very exact model is required, the demand may be changed at every cycle. These formulas may be used to calculate the average order size for each cycle, but this may take a great amount of time. It is, therefore, very impractical to calculate the minimum cost order size quantity at every cycle.

Dividing the total demand by the economic lot size will give the average cycle time. Since the replenishment rate is known, together with the cycle time and the demand for that particular cycle, the behavior of the system can be studied. One of the following will occur:

(a) The system will satisfy the demand and there will be no material remaining in storage to cause overstock. There is, of course, a certain buffer stock remaining in storage.

(b) The system will not meet the demand and stockout will occur. This will cause loss of good will and profit.

(c) The system will be overstocked. There won't be enough demand for the material in storage.

Case (a) is the perfect operation which needs no changes but both cases (b) and (c) require corrections to insure proper operating action. In case (b), the replenishment rate must be increased or, if instantaneous replenishment is used, the lot size must be increased. In case (c), the replenishment rate must be decreased or the lot size diminished.

The decision to increase or decrease the replenishment rate will be made according to the service criterion. Until the system is giving satisfactory customer service, the amount of material to be placed in storage will be varied. A decrease or increase in the cycle time can also be used to adjust the operation of the service. But in this model, only the replenishment rates will be changed, keeping the cycle time constant.

When an order for the increase of the replenishment rate is given, there will be some delays due to lead time. During the transition period between the changes of the replenishment rate, the system may not perform according to the service criterion. The buffer stock will provide the extra material required until the change remedies the situation. Figure 12 is a flow chart for the simulation of an inventory system.

SOLUTION TO THE PROBLEM

Capacitor Network

The capacitor network is almost a perfect analog of the inventory system. The capacitor is made of two conducting plates with a dielectric material between them. When a voltage is applied to the plates, some electrons are forced into one plate, making it negative. Due to the induced positive charges, the other plate becomes positive. A definite amount of electrons are needed to charge a capacitor; therefore, it is said to have a capacity.

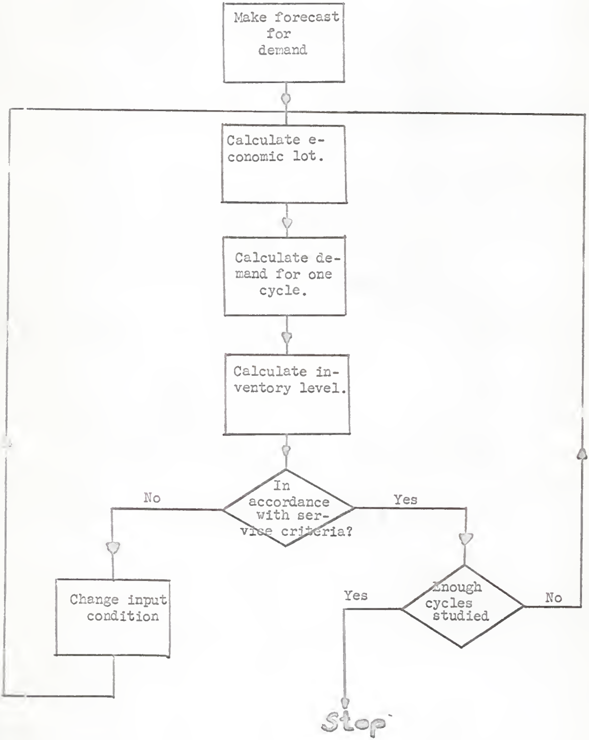


Fig. 12. Flow chart for the simulation of an inventory system.

This characteristic is called capacitance (12).

The negative charges on one plate are attracted by the induced positive charges on the other plate and thus made to stay on the plate. The capacitance of a capacitor is proportional to the quantity of the charge that can be stored for each volt applied across the plates. This is expressed as:

$$C = q/E \qquad \text{(Eq. 9)}$$

where C is the capacitance in farads, q is the charge in coulombs, and E is is the voltage across the plates in volts.

A perfect capacitor with no voltage leakage across the dielectric material should store its charge indefinitely. When uncharged, both plates contain an equal number of free electrons. When charged, one plate contains more charge than the other. The difference is equal to the number of electrons stored in the capacitor.

When a voltage is first applied across the terminals, there is an accumulation of charge which forms a counter voltage across the terminals. The charging continues until this voltage is equal to the applied voltage. By varying the applied voltage, any predetermined amount of charge can be stored. The greater the voltage, the greater the charge on the capacitor. Every capacitor has a limit to the voltage difference between the plates beyond which the capacitor will burst.

When the emf source is replaced by a short circuit in a resistance-capacitance circuit, a discharge current made up of the electrons stored in the capacitor will start flowing through the resistance. As the electrons are removed, the voltage falls rapidly toward zero.

The time required to charge the capacitor to 63% of its maximum voltage or to discharge it to 37% of its voltage is called the time constant of the circuit. Time constant is equal to the product of the value of the resistance in ohms and capacitance in farads. The time constant may also be defined as the time required to charge or to discharge the capacitor completely if the charging continues at its initial rate.

The equation giving the time and voltage relationship in a capacitor is:

$$e_o = E\,(1 - e^{-t/RC}) \qquad \text{(Eq. 10)}$$

where e_o is the instantaneous voltage across the capacitor in volts, E is the applied voltage in volts, t is the time in seconds, and R is the resistance in ohms.

The equation for the charging current is:

$$i_c = (E/R)\,\left(\frac{1}{e^{t/rc}}\right) \qquad \text{(Eq. 11)}$$

When a capacitor is to be used in a particular circuit, the voltage rating and value of the capacitor must be considered.

The capacitor is thus a storing place for electrons; the amount of the charge can be regulated by varying the voltage across the plates. If a resistance is connected across the plates, then the time of the charging and discharging can be varied at will.

As was noted earlier, the capacitor network is almost the perfect analog of the inventory system. If a systematic comparison is to be made, the following points of resemblance must be noted:

(a) A charge placed on a capacitor may be stored there almost indefinitely. In an inventory system, the material placed in storage can be kept there for a long time, assuming that proper conditions for storing that material exist.

(b) The amount of charge that can be stored on a capacitor may be varied at will. The changing of the voltage difference may cause more or less charge to be stored on the plates. This is exactly the same as the inventory system where the amount of material to be stored is varied at will.

(c) There is a maximum amount of charge that can be placed on a capacitor. It cannot accept any more or a breakdown will occur. There is a physical limit to the amount of material that can be stored in a warehouse. Excess material in storage may cause a breakdown of the total system.

(d) The condition of the capacitor depends on the past history of the circuit. Charges that have been stored previously determine how many more can be stored and the amount of charge available for the discharge current. In an inventory system, the amount of material in stock determines how much more can be accepted and how much is available to meet the demand.

(e) The capacitor may be discharged at will. The amount of discharge can be controlled by the resistances in the discharge circuit. Material can be taken out of storage at will and this amount can be controlled in various ways.

(f) By using a high-pass or low-pass filter and different input waves, the charging characteristics of the capacitor may be manipulated. It may be charged instantaneously or at a finite rate. An inventory system may have instant stock replenishment or the replenishment rate may be finite.

(g) The time constant of the capacitor determines the required time for charging or discharging. If proper values of the capacitor and the resistance are chosen, a complete cycle may be represented as a continuous charging and discharging. The charging and discharging may be instantaneous. In an inventory system, the replenishment rate and the rate of demand will determine the cycle time. The replenishment rate or the demand rate may be infinite causing instant replenishment or instant stockout. Both systems are cyclic in behavior.

(h) A capacitor may be charged or discharged for a large number of cycles. In an inventory system, this cyclic behavior will continue as long as it is desired.

(i) The amount of charge on the capacitor may be treated as a discrete variable by calculating the number of electrons. It can also be treated as a continuous variable if the current is measured. It thus represents both the discrete and the continuous inventory systems.

(j) By arranging the circuit parameters, a certain amount of charge may be placed on the capacitor which may be made to retain that particular charge until the required output exceeds a certain value. These charges may then be made to represent a safety stock.

(k) If a delaying circuit can be used, a delay may be introduced between the time that the charging decision is introduced and the time that the actual charging occurs. This is a perfect analog to lead time in the inventory control systems.

(l) If a short circuit connection is provided for instantaneous discharge, then all the charges on the plate are neutralized. This may represent a case of destruction of material in storage due to natural disasters, hurricanes, or even war.

The Analog Model

The twelve similarities between the capacitor network and the inventory system discussed in the preceding section make this network very suitable to represent the inventory system in any physical model. The capacitor represents the storage place and the resistive components of the network represent the flow of materials in and out of storage. Since a systems approach is used, the components that can be used to regulate the input, the output and the feedback and control elements must be used together with the capacitor in order to have the perfect analog model.

Among the system components, the feedback mechanism involves a measure of customer satisfaction. It has already been mentioned that customer satisfaction is extremely hard to measure in quantitative terms. A service criterion was defined which indicated the permitted number of stockouts and thus expressed the extent of customer satisfaction. A physical representation of the feedback mechanism then is very difficult or almost impossible. In the model to be discussed, the feedback mechanism will not be represented by a physical component but will take place through the calculations of the experimenter right after one cycle of operation was studied. This so-called calculation is nothing but a decision about the condition of the inventory level, i.e., whether or not a stockout has occurred. Therefore, one complete cycle has to pass before the experimenter can decide about the feedback information. This is perfectly logical and realistic. In the actual system, one complete cycle has to pass before the feedback information can be used effectively. Thus, if the human link is integrated with the model, the situation will be more realistic since it reproduces the same pattern of feedback action in the actual operation.

Every other system part can be represented either by a single resistor or by a number of basic components. Input will be represented by a pulse obtained from a pulse generator. This input may be regulated by a filter. The regulation of the input will be with respect to the charging time. The amplitude of the pulse will be regulated by the amplitude control of the pulse generator. The output and its characteristic wave shape will be determined by the discharging network. The discharging network will determine the time of the discharge and the length and shape of the discharge wave. A recording device can be connected to the output terminals to record the output wave. This can also be done to the input wave and a complete picture of the input and output conditions will be obtained.

By varying the components of the input and the output networks together with the amplitude of the incoming wave, a wide variety of cases may be simulated and studied.

A short discussion of the input and output networks is appropriate at this point. There are two major classes of networks, each reacting differently to the input waves. These are: (a) low pass networks and (b) high pass networks.

Input waves are also classified according to the order of integration or differentiation operation that must be performed on a reference wave, the unit step function. There are five input waves. These waves and their transient orders are shown in Fig. 13. The step function is taken as the reference point and, if it is integrated once, a ramp function will be obtained. A differentiation carried on a step function will give an impulse. This is known as finite integration and finite differentiation and is true only when the output is very small compared to the input.

High pass and low pass networks act as finite integrators and differentiators. These two networks are shown in Figs. 14 and 15. The response of these networks to various elementary pulses is shown in Figs. 16 and 17.

If a high pass filter is used with unit step input, an output wave, initially having a value of V volts, will be obtained. The value of the voltage will decay exponentially, the length of the cycle being determined by the time constant of the circuit. As mentioned earlier, the time constant equals the product of the resistance in ohms and the capacitance in farads.

An exponential response or output may, of course, be approximated by a straight line over a small section; this is equivalent to considering only small signal deviations around a certain quiescent point of the nonlinear electronic elements. Where the output is so large that the nonlinearity cannot be ignored, it is sometimes possible to define distinct segments on the nonlinear response curve and approximate each segment by a straight line. This is called the piece-wise linear approach (10).

From Fig. 9 it is clear that this network with a unit input may be made to represent an inventory system with instantaneous replenishment. V units are placed in storage at time t_o and will be demanded until time t_T when there will be no more materials in storage, which indicates the end of that particular cycle. The basic principle of operation of the model is regulating the charging and discharging currents of the capacitor by changing the resistance and the capacitance of the circuit with the input. The input and output voltages are then analogous to material flow in and out of storage. By varying resistance R, different demand rates may be obtained. If the results are recorded using a proper scale, a study of the total operation period consisting of many cycles may be made.

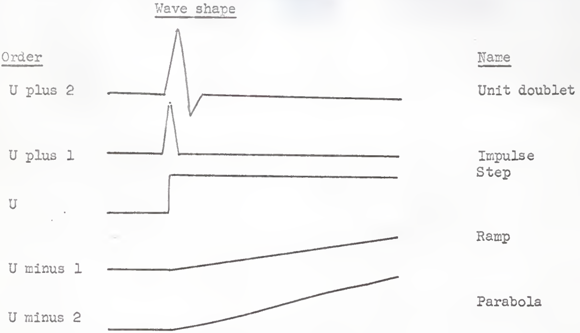


Fig. 13. Transient orders of elementary pulses.

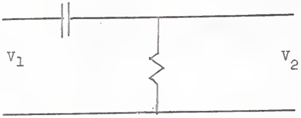


Fig. 14. High pass filter (finite differentiator).

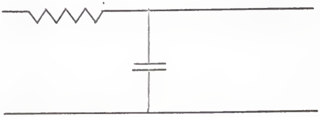


Fig. 15. Low pass filter (finite integrator).

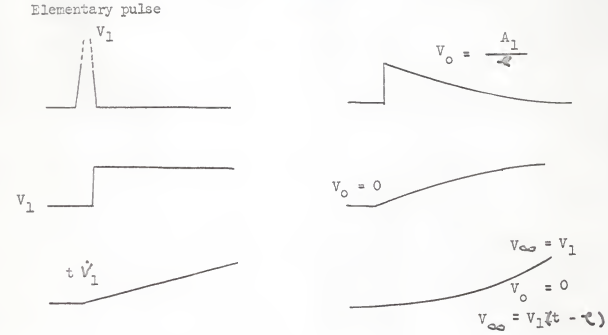


Fig. 16. Output of a low pass filter to inputs of elementary pulses.

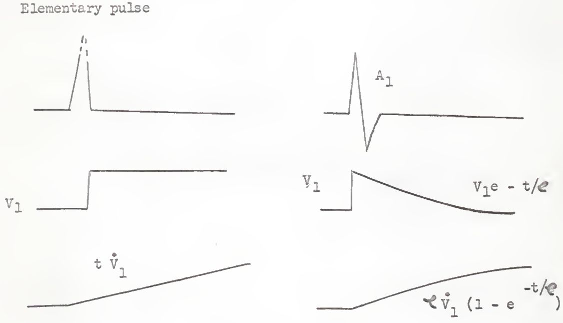


Fig. 17. Output of high pass filter to elementary pulse inputs.

If the demand is varied for each cycle, the total number of stockouts may be read from the recorded results and compared to the service criterion. If the number of stockouts is greater than the service criterion, the amplitude of the input wave (representing the total replenishment) must be increased until the correct replenishment rate is found.

There is a limit to the amount of charge that can be stored in a capacitor. Any increase over this limit will cause the capacitor to burst. The investment and the space constraints may thus be represented by using a capacitor not exceeding a particular value which may be chosen arbitrarily. However, care must be taken to use a single reference capacitor in the experiments.

For example, if the input is V volts indicating V units of material and the capacitor is charged, the voltage difference between the plates will be V volts. When the charging is finished, there will be V units of material in storage and, at time T, the voltage between the plates will depend on the resistance through which the discharge is occurring. In Figs. 18 and 19, two output voltages of a high pass filter are shown, each having a different resistance value. If T is assumed to be the time for a cycle, the output voltage in Fig. 19 reaches zero long before the cycle is finished. In the simulation, this will indicate a case of stockout.

The speed of operation of this model permits a very quick scan of different cases. With the help of a timer, the circuit can be so arranged that a pulse is applied to the input at regular intervals. For example, if the timer is set up for five-second intervals when the cycle time is three seconds with two seconds remaining to change the resistance value, 500 different cases can be studied in 1,500 seconds or approximately in 25 minutes. Notice that no expensive equipment is used. The components used in these models can be found in every laboratory.

The incremental changes in the values of the resistances depend on the sensitivity of the analysis and the scaling. Clearly, if the scaling is such that one volt represents 100,000 units, variations in the value of the resistance in steps of 1 ohm could be tedious and not very realistic. Therefore, reasonable values must be chosen.

One word of caution: the replenishment and the demand curves (those obtained from the analog model) are exponential curves. If the reference axis is so chosen that only the top and the steepest part of the curve are used, the curves can then be approximated by straight lines.

For a finite replenishment case, a discharge resistance is to be provided. Two cases are possible. First, it is assumed that the inventory is to reach its maximum level before any demand occurs. In the model representing this circuit, a switch will be provided between the filter and the output resistance. This switch will be used to connect the network to the discharge resistance only after the charging stops. The switch may be manual or it may be automatic. It senses the voltage across the plates of the capacitor and, when the right value is reached, closes the contact. The input wave is measured at the output terminal of a low pass filter and the output is measured over the resistance provided for discharging. Figure 20 shows the circuit and the input and output functions.

If a circuit as shown in Fig. 20 is used, then the effects of both the input rate and the demand rate variations on the whole circuit can be studied. The same timing circuit can be used to permit a rapid scan of different demand and replenishment rate values. If the cycle time is T, the variations in the rates will give the input-output characteristics shown in Fig. 21.

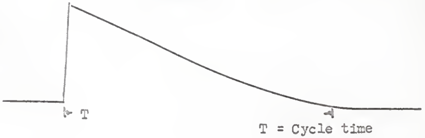


Fig. 18. Output voltage of the analog model, normal operation (HP filter).

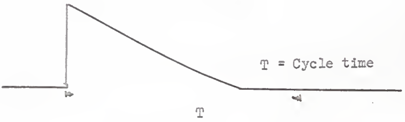


Fig. 19. Output voltage of the analog model, stockout (HP filter).

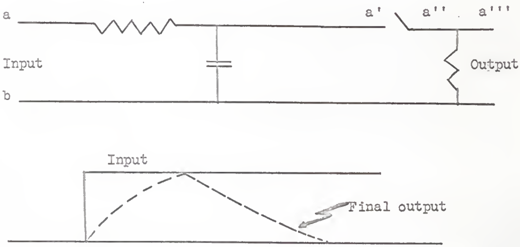


Fig. 20. Low pass filter with an output resistance representing finite replenishment and demand.

There is no need for a switch if the demand is to occur during the replenishment time. Although the capacitor will never charge to its maximum level, a steady state will be obtained, after which the charging will stop and the capacitor will start discharging.

By connecting a high pass and a low pass filter together, the same model may be obtained (Fig. 22).

As can be seen from Fig. 22, only V_3 is to be recorded. The distortion of the waves is more noticeable in this case.

Simulation Using an Analog Computer

The analog model of an inventory system is based on a high pass filter or a high pass-low pass combination. The simulation is carried out by applying an input pulse or an input wave to the filter and then observing the characteristics of the output for different values of the components forming the model. The basic function of the model is finite integration or differentiation.

An analog computer may also be used instead of the filters. Since the main model is based on the high pass filter, which is a finite integrator, an operational amplifier may be used in the same way by using a capacitive feedback between the output and the input. This circuit will integrate all the inputs and the accuracy will be much higher than the high pass filter. Accuracy is not the only advantage of the analog computers. Using the initial conditioning connections, the capacitor may be made to short circuit, removing all residual voltages remaining after the end of the cycle and applying the right voltage as the proper input. A capacitor which is completely discharged at the beginning of each cycle is needed; this is possible only by using short circuiting relays. The step input function may be generated

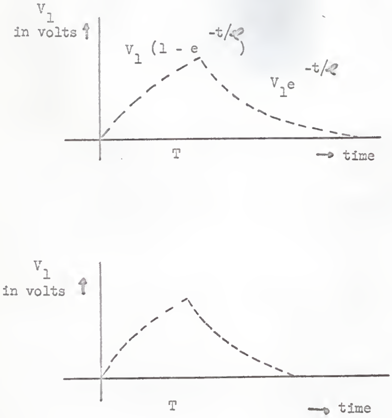


Fig. 21. Variations in the value of the output resistance, a case of normal operation and stockout.

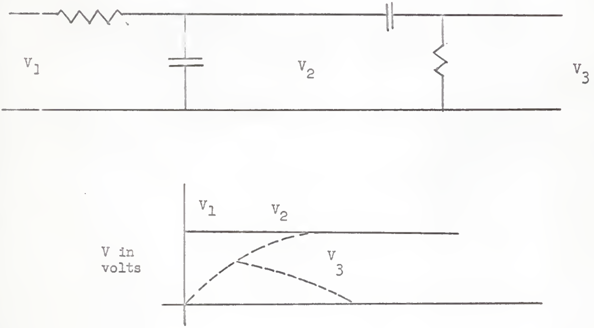


Fig. 22. Combination of a low pass and a high pass filter to represent a case of replenishment during which demand is occurring.

by the computer itself. If the operating mode is set at "Compute," a step voltage is applied at the beginning of each cycle. If any other input wave shapes are desired, they may be produced by using other operational amplifiers. The connections to an outside source create no problems because the output terminals are provided on the computer for recording instruments. If visual readings are sufficient, the voltmeter on the computer may be used. Lastly, it is easier to put together the required circuit on an analog computer than any other place. A filter circuit may create operational problems due to the capacitive and inductive effects between the connections and the components. In analog computers, this effect is minimized by using special conductors.

Three different cases of replenishment were mentioned in previous sections. Therefore, three different models are to be built. One will be used to simulate instant demand. The models to be described can be built on a Heathkit Educational Analog Computer Model EC-1. Any two-channel recorder can be used.

Instant Replenishment Model. A circuit consisting of two operational amplifiers may be used. The circuit connections are shown in Fig. 24. The input to the first amplifier is left floating, i.e., no external voltage source is connected to it. Capacitor C_1 is used to make the operational amplifier function as an integrator. Resistance R_1 is varied in order to change the rate of discharge, corresponding to a change in the demand conditions. Capacitor C_1 is connected to short circuiting relay RL_1. The second operational amplifier is used as an inverter. It performs no operations but inverts the polarity of the output voltage.

When the operation switch is set at "Compute," relay RL_1 will open and a step function will be applied to the input of the first amplifier. This will be integrated according to the following relation:

$$e_o = \frac{1}{R_iC_i}\int e_i \, dt + e_{ic}$$

In this case, e_{ic}, which is the initial condition, is equal to zero. From this equation, it can be seen that variations in C and R will affect the value of the integration.

The second amplifier will invert the output and may multiply it by a given constant. The equation of the output is given by:

$$e_o = -\frac{R_f}{R_i} e_o$$

$$e_o = -\left[\frac{1}{R_iC_i}\int e_i \, dt\right]\frac{R_f}{R_i}$$

After the computation starts, the input will be integrated until the timing circuit interrupts the input function and, at the same time, relay RI_1 will close the short circuit connections. This is the end of one cycle. A one-channel recorder may be used to record the output.

Operating Instructions. The following procedures may be used:

(a) Set up the service criteria.

(b) Calculate the demand limits.

(c) Calculate the economical lot size.

(d) Calculate the approximate number of cycles to be studied during one period.

(e) Set up the integration model and the recorder.

(f) Choose appropriate values for the model components. For example, if R is 100,000 ohms, the demand may be taken as 100 units. The same is true for the capacitor. A 1 microfarad capacitor may represent 100 units of input whereas 10 microfarads may represent 1,000 units. R_2 and R_3 are always to be selected as 1 megohm so that no multiplication effect takes place in the second operational amplifier.

(g) Select the incremental variations of R. If, for example, demand is to be varied by 10 units, then the resistance is to be varied by 100 ohms.

(h) Select the proper cycle times. In the Heathkit computer, time could be varied from 0.1 cycle per second to 15 cycles per second. Therefore, any length of time may be selected for a cycle length with proper scaling. One cycle per second may be made to represent one day. With proper scaling, the real time values for the operation can be obtained from the computer results.

(i) Start simulation by setting the operation control at "Compute" and, after each cycle result is obtained, vary the resistance by a random amount, corresponding to the random variation of demand. This variation may be determined by using a random table or some other predetermined values if the demand distribution is known.

(j) Count the number of stockouts and, if there are more stockouts than the service criteria, increase the value of the replenishment; that is, increase the capacitance value in the model.

(k) Determine the necessary replenishment rate which will insure satisfactory customer service.

Finite Replenishment Model. There are two different cases that can be studied under this heading:

(a) Demand does not occur during the replenishment time, and

(b) Demand does occur during the replenishment time.

Thus, two different models are needed to represent finite replenishment.

No Demand During Replenishment. Under this condition, the maximum inventory level is to be reached before any demand can be satisfied. An analog model for simulation of this particular case is presented in Fig. 20. The same simulation can be carried out on an analog computer. Two operational amplifiers are to be used. The circuit is shown in Fig. 24.

These two amplifiers are connected as operational amplifiers by using capacitive feedback connections. The initial step voltage is applied to the terminals of the first amplifier. The output of this amplifier is the input to the second amplifier. This input is the simulation of the input function to the inventory system. No connections to the short circuit relays are needed and the operation mode is manual. The timing circuit is not used and the cycle time is to be varied by the variations of the components.

When the computer is set to "Compute," a unit step function is applied which is integrated and is then applied to the second amplifier. The second amplifier further integrates this input, thus giving a faithful representation of the replenishment rate. When the capacitor is fully charged, this curve will level off and reach a constant value. Discharge can start any time after this point. This, of course, corresponds to a full warehouse. By setting the operation switch to "Reset," the application of the input voltage is stopped and the capacitor is permitted to discharge slowly, the rate being determined by R and C together. The full recording will show the total relation between replenishment and demand.

The same principle as explained for instant replenishment may be used in this case. The number of stockouts and the overstocks are determined during a given time period and the replenishment rate is adjusted accordingly. The replenishment rate may be changed in the model by changing R_1 or C_1.

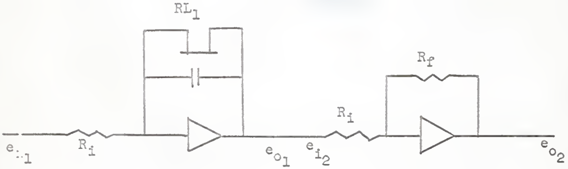


Fig. 23. Instant replenishment model

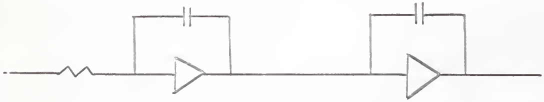


Fig. 24. Circuit to simulate "no demand during replenishment" case.

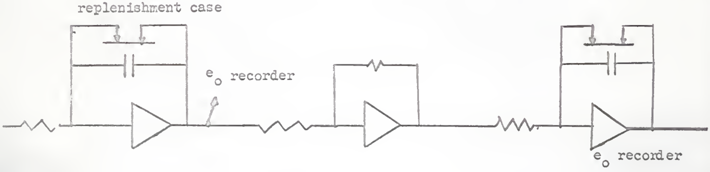


Fig. 25. Circuit to simulate "demand during replenishment" case.

Demand During Replenishment. Three operational amplifiers are to be used in the simulation of the case where demand occurs during the replenishment period. The circuit to be used is shown in Fig. 25.

All capacitors are connected to short circuiting relays. The time of the cycle is to be controlled through the repetition rate control which controls the time during which the input is applied and the time when the capacitors are short circuited. The output of the first amplifier gives the replenishment rate which can be varied through C_1, C_2 and C_3, the most effective variations being through C_1. The demand is varied effectively through R_3, which may be a potentiometer. The output of the third amplifier is the required demand curve. A two-channel recorder is used to record the two outputs. The main difficulty with this model is the scaling. Due to amplification, the scales of the input and the output are not the same. Therefore, the recordings are not directly comparable. When the comparison is to be made, the scale of the output can be determined by the input condition.

This model is very fast in operation and operating conditions are the same as those of the instant replenishment case. The only difference is that the input and the output are not directly comparable.

SUMMARY

Inventory control problems were studied using the systems analysis approach. An analog model was built based on the storage characteristics of a capacitor. Together with the charging and the discharging resistances, eleven points of similarity were found between an inventory system and a capacitor network.

The operation of this model, using various input wave shapes, was made to represent: (1) finite replenishment rate and (2) infinite replenishment rate. By varying the discharge resistance, a great number of demand rates were simulated. The capacitor network used as a model was a finite integrator. It was built on an analog computer using an operational amplifier and a capacitive feedback element. The cycle time was adjusted by timing relays and the study of a great number of cycles in a short time was made possible.

Effectiveness of the system was measured in terms of customer satisfaction which, in turn, was measured in terms of service criteria. Service criteria is the allowable number of stockouts during one period of operation.

During the simulation exercises, the demand was varied randomly and the reaction of the model to these changes was noted. If the total number of stockouts was more than the service criteria, then the input conditions were changed until the system operated in accordance with the service criteria. If the value of the model components was translated into the values of the components of the real system, the most effective system design was obtained.

CONCLUSION

The analog computer will never enjoy the same popularity that the digital computer has. For the majority of people, the analog computers are differential analyzers, extremely useful for solving differential equations but useless otherwise. Yet, some do manage to use general purpose analog computers very successfully. Servomechanism designers can apply analog computers to a wide range of design problems for servo systems.

One common characteristic of modern management is their heavy reliance on digital computers. All sorts of simulation exercises are carried out using them for decision-making purposes. In some cases, the computer is actually forced into a decision-making role. This approach to system simulations and studies tends to destroy the human element and the insight of management.

Since the business world is based on human relations, a full application of the digital computers may have a destructive effect by being forced to play a role beyond their capacity. In the simulation exercises, the digital computer gives almost no information about the actual step-by-step operation of the model. Only the input and the output are given. The computer may be programmed to inform the programmer of every step and every solution but the operation costs make this approach very uneconomical. The results of one complete run must be obtained before the operating characteristics of the model can be changed. Even if more accurate information becomes available during a run, the computer cannot utilize this additional information. It has to be stopped and the calculations made all over again, resulting in an uneconomical situation. For the same reason, variations in the parameters are to be limited to certain values, the amount of variation being determined by the time the computer takes for a single run.

An analog computer, on the other hand, is not plagued by these disadvantages. It is much cheaper to operate, parameters may be varied quickly, a total picture of the operation may be obtained easily and, at any point during the computation, human intervention can be exercised. The most recent information can be made available to the computer almost at once.

The model discussed in this article for the inventory systems has all these favorable characteristics. The most useful of these are the total picture of the operation which it presents to the analyst and the link between cycles which is to be provided by the operator. When the total picture is available, it is very easy for the operator to use common sense and eliminate certain inputs as unfeasible. Thus, human intuition and foresight are not eliminated from the model but are a part of it. The analyst is not a passive observer but an active link that gains knowledge and first-hand experience about model characteristics.

The process of building the analog model may help discover new characteristics of the system. Sometimes a new approach can be found while building an analog model which may result in a better system design.

Operation of analog computers is limited to continuous cases which prevents their application to discrete cases. Even for a continuous case there is no clear method of approach to the problem. A systems approach is very helpful but after the system characteristics, boundaries and constraints are found, the next step may require some unorthodox thinking. For some servo components, basic analog models have been set up. The same is true for many transfer functions. There is a good possibility for having a common model for a certain problem class.

Even after building an analog model, many people prefer to use a special analog computer instead of the general purpose analog computer. Building a special purpose computer requires a thorough knowledge of the basic principles of mechanics and electronics. Problems of transportation have been solved on special analog computers. Although the method is well-known, not many people use this approach because building a special purpose computer is not very economical.

This study was an effort on the part of the investigator to explore possibilities of applying general purpose analog computers to a specific problem. The ease with which the model can be built and simulated and the results interpreted caused the author to feel that great possibilities do exist in the application of these computers to a variety of problems. The problems studied were rather heavily restricted to certain cases. These limitations did not prevent a realistic approach; in fact, they only limited its use. Similar limitations also exist in mathematical models.

Further studies into the nature of the problem are desirable. A common model list may be drawn up for business and manufacturing use. If an inventory control, purchasing and production control systems can be represented as analog models, the study of the total business and manufacturing systems becomes a possibility using analog computers.

Industrial engineers may gain an extremely valuable tool with more research, thus giving themselves the opportunity to study man-machine systems more effectively.

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APPLICATION OF ANALOG COMPUTERS TO INVENTORY CONTROL PROBLEMS

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B. S. E. E., Robert College, Istanbul, 1964

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

ABSTRACT

When the systems analysis technique is applied to inventory control problems, it is possible to formulate an approach which permits the building of an analog model and the simulation of the system using the analog computer.

The first step in this technique is to determine the system components: 1. Input - the material placed in storage; 2. Output - the material used to satisfy the demand; 3. Processor - the combination of techniques and methods used to regulate the flow of material in and out of storage; 4. Feedback mechanism - the customer satisfaction; and 5. Control mechanism - the financial and other constraints that limit the amount of material to be stocked.

Effectiveness of the operation of the inventory system can be expressed in terms of customer satisfaction. Since it is difficult to measure satisfaction in quantitative terms, an empirical means of representation, called service criterion, is used and is defined as the permitted number of stockouts during one period of operation. If the number of stockouts is greater than the service criterion, then the service is not satisfactory. The formula for service criterion is:

$$S.C. = L/S$$

where S. C. is the service criterion, L is the loss in dollars that is acceptable by the firm due to stockouts and S is the dollar cost of one stockout.

Service criterion plays a very important role in the feedback mechanism of the system. When the actual operation is simulated by the model, the number of stockouts can be determined and the service criterion calculated. The input conditions may be changed so that the service criterion is satisfied. This, of course, is the point where the system is giving maximum satisfactory customer service under the given conditions and constraints.

An analog computer with a capacitor network satisfactorily simulates an inventory control problem. Together with the charging and discharging networks made out of parallel or series resistances, the analog computer can be made to represent an inventory control system under every operation condition. The capacitor is a storing element and, working with the charging and the discharging networks, it acts as a finite integrator.

A model built on an analog computer permits the simulation of a period of operation consisting of many cycles. Sales and stocking conditions may be varied at will.

In the analog computer simulation, demand is varied between two limits and the response of the system to changing demand conditions is noted. If necessary, input conditions may be changed so that the system operates according to the desired service criterion, thus providing maximum customer satisfaction.